DI Physics Hons.

Paper - I

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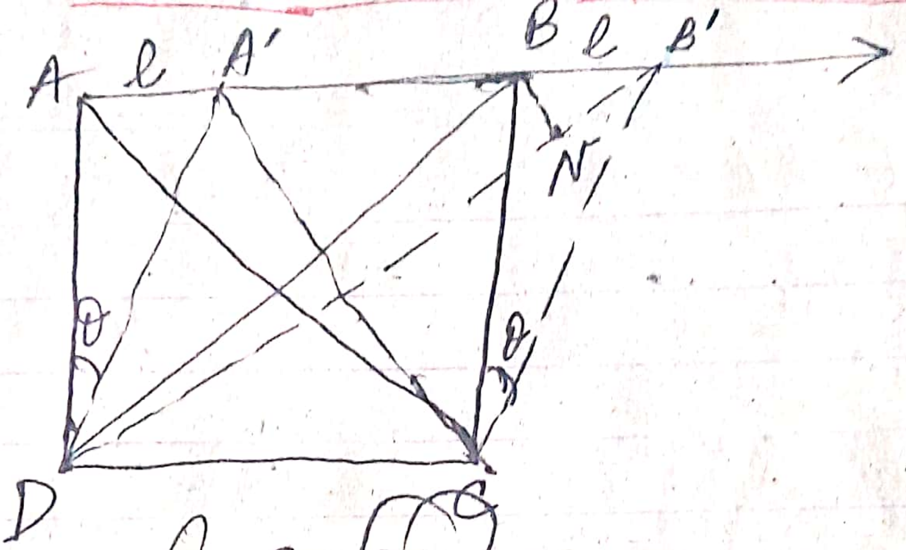

fig. 2.

Let ABCD represent the front face of a cube of side L. A tangential force F is applied on its upper face AB and the bottom face DC is fixed. As a result of this force the cube is sheared to A'B'CD through an angle. Then, shearing strain

$$\theta = \frac{AA'}{AD} = \frac{BB'}{BC} = \frac{l}{L},$$

where the displacement

$AA' = BB' = l$.

Shearing stress

$$T = \frac{F}{\text{area of the upper face of the cube}} = \frac{F}{L^2}$$

$\therefore$ coefficient of rigidity $n = T/\theta$.

But a shearing stress along AB is equivalent to a tensile stress along DB and an equal compressive stress along AC at right angles to each other.

Let α and β be the longitudinal and lateral strains per unit stress respectively. The extension along diagonal DB due to tensile stress

$$= DB \cdot T \cdot \alpha$$

and extension along diagonal DB due to compression stress along AC

$$= DB \cdot T \, \beta.$$

Total extension along DB

$$= DB \cdot T \cdot (\alpha + \beta) = L\sqrt{2} \cdot T \cdot (\alpha + \beta)$$

Let us draw a perpendicular BN on DB'. Then increase in the length of diagonal DB is practically equal to NB'. As θ is very small, therefore angle AB'C is nearly 90° and $\angle BB'N = 45^\circ$.

Thus, $NB' = BB' \cos 45^\circ = \dfrac{BB'}{\sqrt{2}} = \dfrac{l}{\sqrt{2}}$,

$$\therefore L\sqrt{2} \cdot T(\alpha + \beta) = l/\sqrt{2}$$

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or, $T \cdot \frac{L}{\ell} = \frac{1}{2(\alpha+\beta)}$

But $T \cdot \frac{L}{\ell} = \frac{T}{\ell/L} = \frac{T}{\theta} = n$,

$\therefore n = \frac{1}{2(\alpha+\beta)} = \frac{1}{2\alpha\left[1+\beta/\alpha\right]}$.

But $\frac{\beta}{\alpha} = \sigma$ and $Y = \frac{\text{Stress}}{\text{longitudinal strain}} = \frac{1}{\alpha}$

[$\because \alpha$ is the longitudinal strain per unit stress.]

Therefore, $n = \frac{Y}{2(1+\sigma)}$ --- (ii)

(iii) Relation between Y, K and n.

From relation (i) $1 - 2\sigma = Y/3K$

From relation (ii) $2 + 2\sigma = Y/n$

Adding the above two equations, we get

$$3 = \frac{Y}{n} + \frac{Y}{3K} = Y\left(\frac{1}{n} + \frac{1}{3K}\right)$$

$$= Y\left(\frac{3K+n}{3nK}\right)$$

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Thus, $Y = \dfrac{9\eta K}{3K+\eta}$ ---- [iii (a)]

This may be written as

$$\frac{9}{Y} = \frac{3K+\eta}{\eta K}$$

or, $\dfrac{9}{Y} = \dfrac{3}{\eta} + \dfrac{1}{K}$. ---- [iii (b)]

(iv) Relation between K, η and σ:

From relations (i) and (ii), we have

$$Y = 3K(1-2\sigma)$$

and $Y = 2\eta(1+\sigma)$

Equating the two values of Y, we have

$$3K(1-2\sigma) = 2\eta(1+\sigma)$$

or, $3K - 6K\sigma = 2\eta + 2\eta\sigma$

or, $\sigma(2\eta + 6K) = 3K - 2\eta$

Thus;

$$\sigma = \frac{3K-2\eta}{2\eta+6K}$$ ---- (iv)